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SVD Algorithm for Lossy Image Compression

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Abstract:- **Singular Value Decomposition as per it could be defined is the method of factorizing the real/complex matrix. These days the SVD algorithm tends to play a vital role in Image Processing. Its extending applications prove to be of a greater use. Not just image processing it also has equal importance in Statistics. It is the transformation of values in three different methods. In these different methods also the image compression can be done only through small set of values. The main objective of this methodology is to achieve this image compression using very less storage space. And also to preserve the most important set of values related to the image. The implementation is done through MATLAB.**

For example if m*n is a matrix
SVD of A can be explained as the equation of A=UΣVT
where, U is the product of m and m and orthogonal V is also the product of n and n and orthogonal and Σ is m*n diagonal matrix with all the positive set of values.

Keywords:--**Image Processing, Orthogonal, MATLAB, Diagonal matrix, Real and Complex matrix**

I. INTRODUCTION

Developments has reduced the work to store the digital information that has to be generated or transmitted. Image compression is the exploitation or reduction of the acquired band-width in order to manage the storage. Not just the storage capacity it also enhances the rate of transmission. The main advantage is to reduce the percent of redundancy and irrelevancy.[6] Image compression also enables reconstruction. The degree of compression is determined by the digital information contained in the image. This technique which requires SVD algorithm is also used for the process of biometric systems and for the process of face recognition techniques.

The transformation of image processing techniques can be exploited at the storing and retrieval of a digital image[5]. The main application for this image compressing technique can also be applied in the fields of satellite imaging process, medical imaging, and also the image enhancement.[2] With the growing trends of technology through high speed computers, signal processors, image processing has become easier through the digital processing procedures.[1] Image processing takes place in the following order:

1.SVD algorithm on the system tests for refactoring of the message signal.

2.Using the different values for K image reconstruction for the input images are performed.

3. The computational values for values of Compression Ratios(CR), Mean Square Error and Peak to Noise signal ratios for both the quality and quantity measurement of the image that is compressed, as per the measure of the compression in the standards of performance.

4.As per the varying values of K the performance in variation is evaluated.

II. IMAGE COMPRESSION

As per the algorithm designed the image can be represented using matrix representation in the form of (m*n), in which m represents the number of rows and also the pixel height of the images while n is the number of columns and also the pixel width of the image. Any picture represents its darkness and brightness value when stored in a computer with a specific value that is assigned in the form of a number.[5] The brightness of the corresponding image is represented through the values in the matrix. In case of any gray scale image, the range of values are 0-black and 1-white within the matrix equally represents the image as light or dark.[3] The computer layers-out the images into 3 layers namely red, blue and green in the images and for any colored images the spaced occupied is 7% more than actual size. Again the same single-colored picture is compared to the grayscale picture for the measurement of darkness and the brightness factors, which is further recombined and reconstructed to form the original or the input image.

In every color image, the color pixels are classified into 3 primary components namely red, blue and green as per discussed.[9] And all these pixels ranged from

0 (no color) to 1(saturated image). For instance of a 9 mega-pixel of gray scaled image could be represented 3000 x 3000 pixels of matrix.[9] Certain integer value ranging from 0-255 is used for representation of the same gray scale image through the matrix values.[9] As per the calculations every store image requires 9MB of space which requires byte of space per every pixel. For the colored image as the storage value is high storing color images is 3 times the actual size that is 27MB as it consists three different color components.[7]

III. REDUNDANCIES

There exists three kinds of redundancies through the system implemented:

A. Coding Redundancy

For usage of optimal code words used is less than the required, there occurs Coding Redundancy. In many cases Look Up Tables(LUTs)[3] are the basic source for the implementation of the SVD algorithm and makes the coding techniques reversible.[8] The most exercised image coding schemes are Huffman and Arithmetic coding schemes.

B. Inter pixel Redundancy

Based on the values of other corresponding pixels, the pixel value can easily be predicted. This redundancy occurs mainly due to mapping of 2 dimensional array pixels into several other formats. This methodology is also called as Spatial Redundancy/ Inter frame / Geometric redundancy.[3]

C. Psycho-visual Redundancy

As per the studies of Psycho-Physical department the human eye is sensitive to several band of frequencies that is human eyes cannot receive all the frequencies with the equal sensitivity. Some parts of the information come to be of more importance than others. Due to this, there is another type of redundancy that comes into existence, which is called Psycho-visual redundancy[3].

IV. SINGULAR VALUE DECOMPOSITION

Singular Value Decomposition (SVD) deals with the decomposition of general matrices which has proven to be useful for numerous applications in science and engineering disciplines. The SVD is commonly used in the solution of unconstrained linear least squares problems, matrix rank estimation and canonical correlation analysis. Computational science exploits SVD for information retrieval, seismic reflection tomography, and real-time signal processing[2][7].

A. Theory

The goal of SVD is to find the best approximation of the original data points that is of large dimensions, using fewer dimensions. This is possible by identifying regions of maximum variations. So when a high dimensional, highly variable set of data points is taken, SVD is employed to reduce it to a lower dimensional space that exposes the substructure of the original data more clearly and orders it from most variation to the least. In this way, the region of most variation can be found and its dimensions can be reduced using the method of SVD. In other words, SVD can be seen as a method for data reduction. [3] The singular value decomposition is defined as a factorization of a real or complex, square or non-square matrix.

Consider a matrix A with m rows, n columns and rank r[2].
Then A can be factorized into three matrices:

$$A = U\Sigma VT$$

(1) [8] Here U and V are ortho-normal matrices and the matrix Σ is a diagonal matrix with positive real entries [7]:
U is an m × m orthogonal matrix
VT is the conjugate transpose of the n × n orthogonal matrix [7].
Σ is an m × n diagonal matrix with non-negative real numbers on the diagonal which are known as the singular values of A [7].
The m columns of U and n columns of V are called the left-singular and right-singular vectors of A respectively.
The singular values of Σ are arranged as σ1□σ2□…□σr□0, [7] where the largest singular values precede the smallest and they appear on the main diagonal of Σ.
The numbers σ12 ≥ … ≥ σr2 are the Eigen values of AAT and ATA. [7]

B. Steps to calculate SVD of a matrix

1) First, calculate AA^T and A^TA.
2) Use AAT to find the Eigen values and eigenvectors to form the columns of U: $(AA^T - \lambda I)\ddot{x} = 0$ [3].
3) Use ATA to find the Eigen values and Eigen vectors to form the columns of V: $(A^TA - \lambda I)\ddot{x} = 0$.
4) Divide each eigenvector by its magnitude to form the columns of U and V.
5) Take the square root of the Eigen values to find the singular values, and arrange them in the diagonal matrix S in descending order: σ1 ≥ σ2 ≥ … ≥ σr ≥ 0 [3].

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C. Properties of SVD

Many properties of SVD are listed as follows:

1) The singular values 'σ' are unique, unlike, the matrices U and V, which are not unique.[1]

2) The singular values of a rectangular matrix A are equal to the square roots of the Eigen values λ1, λ2 . . . λm of the matrix A^TA.[1]

3) Mathematically, the rank of the matrix A is the number of its non-zero positive singular values; rank (A) = r, r ≤ m.

4) Since $AA^T = U\Sigma(T)U(T)$, so U diagonalizes AA^T and u(i) s are the eigenvectors of AA^T .

5) Since $A^TA = V\Sigma(T)\Sigma V(T)$, V diagonalizes A^TA and the v(j)s are the eigenvectors of A^TA [1].

6) If A has rank 'r' then vj, vj, … ,vr form an ortho normal basis for range space of A^T , $R(A^T)$, and uj, uj, … ,ur form an orthonormal basis for range space A, R(A)[1].

D. SVD Approach to Image Compression

SVD divides a square matrix into two orthogonal matrices(U,V) and a diagonal matrix (∑).So the original matrix is rewritten as a sum of much simpler rank-one matrices. SVD is applied on an image matrix A to decompose it into 3 different matrices U, ∑ and V. But applying SVD alone does not compress the image. To compress an image, after applying SVD, only a few singular values have to be retained while other singular values have to be discarded . All the singular values are arranged in descending order on the diagonal of ∑matrix [1].

The discarding of the SV's follows the fact that the first singular value on the diagonal of ∑contains the greatest amount of information and subsequent singular values contain decreasing amounts of image information [1]. Thus, negligible amount of information is contained in the lower SV's. So, they can be positively discarded after performing SVD, simultaneously avoiding significant image distortion [1].Furthermore, property 3 of SVD (section V 'C') says that 'the number of non-zero singular values of A is equal to the rank of A'. In cases where the lower order singular values after the rank of the matrix are not zero, the discarding can still be done since they have negligible values and are treated as noise [1].

SVD image compression process can be illustrated by implementing the following algorithm, where a given matrix A is expressed as follows:

E. SVD Image Compression Measures

To measure the performance of the SVD compression, the quantitative and qualitative measurement of the compressed image is found by calculating the following :

1. Compression Ratio (CR) Compression Ratio is defined as the ratio of file sizes of the uncompressed image to that of the compressed image[4]:

2. Mean Square Error (MSE) MSE is defined as square of the difference between pixel value of original image and the corresponding pixel value of the compressed image averaged over the entire image[1].

Mean Square Error (MSE) is computed to measure the quality difference between the original image A and the compressed image A^k.

3. Peak Signal to Noise Ratio (PSNR) Peak signal-to-noise ratio (PSNR) is defined as the ratio between the maximum possible power of a signal and the power of corrupting noise that affects the fidelity of its representation. PSNR is usually expressed in terms of the logarithmic decibel scale to accommodate signals with a wide range. In lossy compression, the quality of compressed image is determined by calculating PSNR. The signal in this case is the original data, and the noise is the error introduced by compression. The PSNR (in dB) is given by the equations [3]:

- START
- Read the input image
- Convert integer to double data type
- Calculate the required rank 'k'
- Perform SVD to obtain U, ∑ , V matrices
- Apply approximation on ∑ matrices
- Regenerate the matrix and remove singularity
- Convert double data type to integers
- Compressed image is created and displayed
- STOP

DIFFERENT VALUES OF 'K'

[Bytes of input image: 124358]

STEP I:
Load the JPEG image
STEP II:
Store image as array of integers
STEP III:
Specify Compression Ratio
STEP IV:
Perform SVD on matrix components
STEP IV:
Regenerate decomposed matrix
SEP V:
Remove singularity if any
STEP VI:
Collect matrix components
STEP VII:
Create the compressed image

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No. of Singular Values used 'k'
Performance Evaluation Parameters Compression Ratio (CR)
MSE (in dB)
PSNR (in dB)
Bytes of Compressed Image (B)
2 61.046 40.61 25.04 30796 12 23.588 39.42 28.17 31679 22 12.101 38.09 30.32 31699 52 5.72 38.047 32.39 31781 112 2.01 37.44 33.46 31815 R=202 0.580 -20.93 83.88 31840 251 0.564 -20.72 86.42 31862 260 0.552 -20.69 86.437 31905 262 0.544 -20.66 86.453 31926 264 0.536 -20.66 86.453 32124

V. EXPERIMENT AND RESULT ANALYSIS

Initially the processor is fed with the JPEG image which has to be compressed [8]. This input image is stored as an array of integers. For performing compression in MATLAB, the array of integers is converted to double data type. Now, prior to the compression process, the user needs to specify the amount of compression that is desired. This is achieved by specifying the Compression Ratio [8] for the particular input JPEG image. Singular Value Decomposition is performed to re factor the input image matrix and is then applied separately to the matrix components. The resultant decomposed matrix is regenerated by decoding the bit stream. SVD compression technique is applied to the input image for different singular values and the compressed images are created. Considering different values of k, the result of SVD compression is depicted in the images displayed above. The displayed image is extremely blurred when the singular value is chosen to be 3,.Alternatively, it means that only first 2 eigen values of ∑ matrix are considered for image reconstruction. The image obtained by applying SVD using k=12 as the singular value.

Following inference can be drawn on the basis of the above table and plots:

1. The Eigen values used in the reconstruction of the compressed image are represented by the parameter 'k'.
2. Smaller values of k imply greater compression ratio (i.e. less storage space is required) but a deterioration in the image quality (i.e. larger MSE values& smaller PSNR values).
3. Thus, it is necessary to strike a balance between storage space required and image quality for good image compression. From the above observations, it is found that optimum compression results are obtained when MSE of the compressed image is just less than or equal to 38dB (i.e. MSE ≤ 38dB). In our case, this is obtained when value of k is 52.
4. Thus, it is necessary to strike a balance between storage space required and image quality for good image compression. From the above observations, it is found that optimum compression results are obtained when MSE of the compressed image is just less than or equal to 38dB (i.e. MSE ≤ 38dB). In our case, this is obtained when value of k is 52.
5. When k is equal to the rank of the image matrix (202 here), the reconstructed image is almost same as the original one. And as k is increased further, there is a negligible decrease in the MSE values. This means that improvement in the image quality is very negligible.

VI. CONCLUSION

In performing SVD compression for JPEG images the values of Compression Ratio and their variation with corresponding singular values (SVD coefficients) are observed and their relation is concluded to be a decreasing exponential function. More compression ratio can be achieved for smaller ranks. On the other hand, the computation time for the compressed images is the same for all the values of k taken. It was also found that the fewer the singular values were used, the smaller the resulting file size was. An increase in the number of SVD coefficients causes an increase in the resulting file size of the compressed image. , As the number of SVD coefficients nears the rank of the original image matrix, the value of Compression Ratio approaches one. From the observations recorded it can be seen that the Mean Square Error decreases with increase in the number of SVD coefficients, unlike PSNR which varies inversely with the value of 'k'. Therefore, an optimum value for 'k' must be chosen, with an acceptable error, which
(a)k=2
(b)k=12
(c)k=22
(d)k=52
(e)k=112
(f) k=202
(g)k=251
(h)k=260
(i)k=262
(j)k=264
conveys most of the information contained in the original image, and has an acceptable file size too.

Applications
1. An optimum value of Compression Ratio (CR) is characteristic to any image compression technique to make

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the compressed image well adapted to statistical variations [2]. SVD has proven to be advantageous in this aspect.

2. The range and null space of a matrix are important quantities in linear algebraic operations, which are explicitly defined by SVD, through the left and right singular vectors (U & V resp.). Vector U has vanishing singular values of original image that span its 'null space'. Vector V contains the non-zero singular values of original image that span the 'range'.

3. Noise reduction is also one of the many applications of SVD. In this paper A stands for an image matrix. Likewise, if A represents a noisy signal, then on computation of SVD, small singular values of A can be discarded. The discarded SV's mainly represent noise. Hence, the compressed signal Ak represents a noise filtered signal.

4. SVD also finds its application in the area of Face Recognition

Future work

The future work will focus on the use of Wavelet Difference Reduction (WDR) and Adaptively Scanned Wavelet Difference Reduction (ASWDR) for further compressing of the image. WDR offers high compression of the overall system. ASWDR adapts the scanning procedure used by WDR in order to predict locations of the significant transform values at half thresholds. The study will also focus on image reconstruction and face recognition.

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